Journal of Applied Mechanics and Technical Physics, Vol. 49, No. 2, pp. 201–208, 2008

HYDRODYNAMIC ANALYSIS OF THE DISPLACEMENT CONDITIONS OF FORMATION FLUIDS USING AN AXISYMMETRIC MODEL

I. B. Chernoshchuk

UDC 532.546

The axisymmetric problem of the displacement of formation fluids by a drilling mud filtrate with filter cake formation is considered. An analysis is made of the distribution and variation of the main parameters of the process: filtrate volume, filter cake thickness, oil saturation, and pressure. The positions of the water-saturation and salt-concentration fronts are determined. The results are compared with the geophysical logging data for straight-hole drilling.

Key words: *invaded zone, oil saturation, salt concentration, pressure, filtrate volume, filter cake.*

Introduction. Borehole drilling leads to a high pressure difference, because of which the drilling mud filtrate invades the oil-filled formation and displaces the formation water and oil, thus causing a change in the physical properties of the medium. A hydrodynamic analysis of these processes based geophysical logging data for the near-borehole region provides information on the hydrophysical characteristics of the formation. As a rule, the axisymmetric problem is solved as a first step in the analysis of oil-saturation and salt-concentration distributions. The data obtained in this step allow more accurate studies using spatial models to be performed.

Hydrodynamic Model. Let us consider the problem in a one-dimensional axisymmetric formulation (the symmetry axis coincides with the borehole axis). Two-phase filtration is described by the Buckley–Leverett equations under the additional condition imposed on the sum of the water and oil saturations. The transport equations for the moving phases are written as [1–3]

$$\frac{\partial}{\partial t}(rms_{\rm w}) = \frac{\partial}{\partial r}\Big(rk_{\rm w}\,\frac{\partial p}{\partial r}\Big); \tag{1}$$

$$\frac{\partial}{\partial t}(rms_{\rm oil}) = \frac{\partial}{\partial r}\Big(rk_{\rm oil}\,\frac{\partial p}{\partial r}\Big); \tag{2}$$

$$s_{\rm w} + s_{\rm oil} = 1, \tag{3}$$

where r is the radius ($r_b \leqslant r \leqslant L$), r_b is the borehole radius, L is the boundary of the region (the radius of influence), t is time, p is the difference between the true formation pressure and the initial formation pressure p_f, $s_{\rm w}$ and $s_{\rm oil}$ are the water and oil saturations, m is the formation porosity, $k_{\rm w}(s_{\rm w})$ and $k_{\rm oil}(s_{\rm oil})$ are the relative permeabilities. The boundary conditions on the borehole wall ($r = r_b$) are specified as follows:

— for water-saturation,

$$s_{\rm w}\Big|_{r=r_b} = 1; \tag{4}$$

— for pressure [4],

$$-q = \Big(k_{\rm w}\,\frac{\partial p}{\partial r}\Big)\Big|_{r=r_b} = \beta\Big(p\Big|_{r=r_b} - p_b(t)\Big), \qquad \beta = \Big(\beta_0^{-1} + \frac{d}{k_g^0}\Big)^{-1}. \tag{5}$$

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090; irina@hydro.nsc.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 49, No. 2, pp. 56–64, March–April, 2008. Original article submitted June 21, 2004; revision submitted May 18, 2007.

The parameter β defines the water exchange between the borehole and the formation and takes into account the flow coefficient at the initial moment of formation drilling β_0^{-1} and the resistance d/k_g^0 due to filter cake formation ($k_g^0 = k_g/\mu w$, where k_g is the filter cake permeability and μ_w is the water viscosity). The growth rate of the filter cake is proportional to the filtrate volume (transport of clay particles to the formation and porosity variation in the borehole zone are ignored); therefore, the filter cake thickness $d(t)$ is defined by the equation

$$d'_t = \alpha(1 - d/d_{\max})^n q, \qquad n \geqslant 0. \tag{6}$$

On the boundary of the region, the pressure obeys the condition

$$p|_{r=L} = 0. \tag{7}$$

In Eqs. (5) and (6), $p_b(t)$ is the specified borehole pressure, $d_{\max}$ is the maximum thickness of the filter cake, α is a coefficient that depends on the porosity of the filter cake, the fraction of clay particles in the drilling fluid, and the other parameters determining the filter cake growth conditions [2]. The initial water-saturation distribution is constant:

$$s_{\mathrm{w}}|_{t=0} = s_f. \tag{8}$$

We note that Eqs. (1) and (2) are written for moving phases. If the effect of the residual water and oil saturations is taken into account, it is necessary to consider the general equations, which, in turn, can be reduced to the form of (1), (2 [5].

In the case of an incompressible layer ($m = \mathrm{const}$), Eqs. (1) and (2) can be transformed. Combining these equations with allowance for condition (3) leads to the following expression for the total circulation rate:

$$r(k_{\mathrm{w}} + k_{\mathrm{oil}})\,\frac{\partial p}{\partial r} = -v(t). \tag{9}$$

We denote $s = s_{\mathrm{w}}$ and introduce the function

$$F(s) = \frac{k_{\mathrm{w}}(s)}{k_{\mathrm{w}}(s) + k_{\mathrm{oil}}(s)}. \tag{10}$$

Then, from Eqs. (9) and (10), we obtain $-v(t)F(s) = rk_{\mathrm{w}}(s)\,\partial p/\partial r$. Substitution of this relation into (1) yields

$$rm\,\frac{\partial s}{\partial t} = -v(t)\,\frac{\partial F(s)}{\partial r}. \tag{11}$$

Let us transform to the self-similar variable $y = (r^2 - r_b^2)\Big/\Big(2\int\limits_0^t v(t)\,dt\Big)$. Then, Eq. (11) becomes $my = dF/ds$ or $m(r^2 - r_b^2)\Big/\Big(2\int\limits_0^t v(t)\,dt\Big) = dF/ds$. The boundary condition (4) is satisfied if $(dF/ds)|_{t=r_b} = 0$. Denoting the specific filtrate volume by $V(t) = \int\limits_0^t v(t)\,dt$, we finally obtain

$$\frac{m}{2}\,\frac{r^2 - r_b^2}{V(t)} = \frac{dF}{ds}. \tag{12}$$

Equation (12) allows one to find the distribution of the water saturation s for any value of V. In the calculations, we assumed that the quantity s is specified on the grid ($s_f \leqslant s \leqslant 1$) and sought the corresponding values of $r = \sqrt{r_b^2 + (2V(t)/m)\,dF/ds}$. One should bear in mind that, first, it is necessary that the distribution s be unique, and, second, that, beginning from a certain distance from the borehole axis $r = l_s$, the saturation remain constant and equal to the initial formation-water saturation s_f (8). Therefore, the dependence $s(r)$ was determined as follows: for $r < l_s$, the parameter s was determined from formula (12), and for $r \geqslant l_s$, $s = s_f$.

Let us find the value of $r = l_s(t)$. Integrating Eq. (11) first over the radius r from r_b to L, and then over time t from 0 to t, we obtain

$$\int\limits_{r_b}^{L} rm(s - s_f)\,dr = -(F(s_f) - 1)\int\limits_0^t v(t)\,dt = (1 - F(s_f))V(t).$$

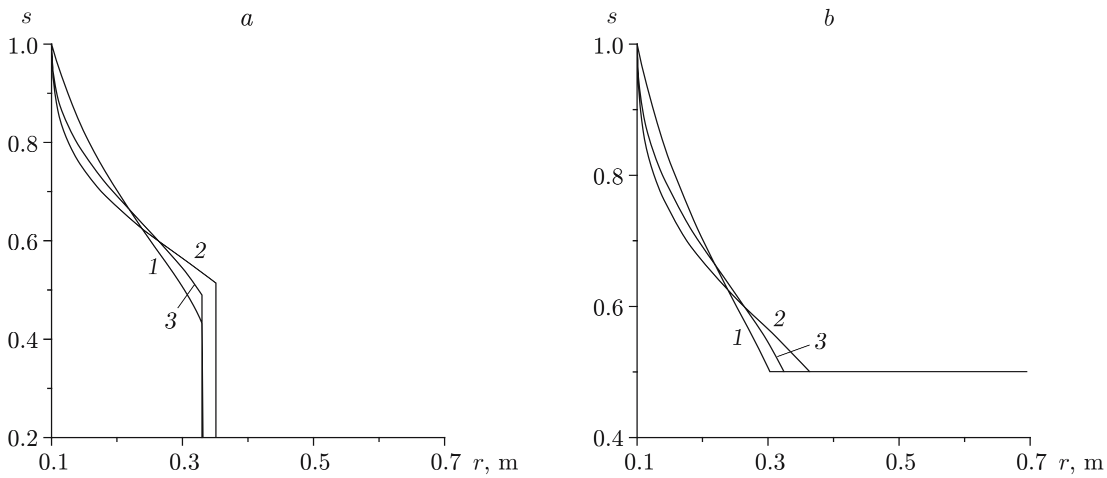

Fig. 1. Water-saturation distribution along the radius for various phase permeabilities: $s_f = 0.2$ (a) and 0.5 (b).

Because, $s = s_f$ for $r \geqslant l_s$, it follows that

$$\int_{r_b}^{l_s} rm(s - s_f)\,dr = (1 - F(s_f))V(t). \tag{13}$$

The integral in (13) is calculated using the trapezoid method. The summation over Δr is performed until the inequality sign between the right and left sides is reversed. Thus, the quantity $r = l_s$ is determined. Figure 1 shows the distribution of s along the radius for a fixed specific (per unit length of the borehole) filtrate volume $V(t) = 0.005$ m^2 and formation water saturations $s_f = 0.2$ and 0.5 ($r_b = 0.1$ m, $\mu_0 = 0.4$, $m = 0.2$, $k = 1$ millidarcy $\approx 1.02 \cdot 10^{-15}$ m^2, $\mu_w = 1$ cP $\approx 10^{-3}$ Pa $\cdot$ sec. Curves 1 and 2 differ by the dependences $k_{\rm w} = k_0 s^{n_{\rm w}}$ and $k_{\rm oil} = k_0\mu_0(1 - s)^{n_{\rm oil}}$ ($k_0 = k/\mu_{\rm w}$, where k is the formation permeability and $\mu_0 = \mu_{\rm w}/\mu_{\rm oil}$ is the ratio of the water and oil viscosities): for curves 1, $n_{\rm w} = n_{\rm oil} = 2$, and for curves 2, $n_{\rm w} = n_{\rm oil} = 3$. Curves 3 correspond to a more universal form of the relative permeability relations: $k_{\rm w} = k_0[\delta s^{\alpha_1} + (1-\delta)s^{\beta_1}]$ and $k_{\rm oil} = k_0\mu_0[\delta s^{\alpha_2} + (1-\delta)(1-s)^{\beta_2}]$, where $0 \leqslant \delta(s) \leqslant 1$. In this case, $\alpha_1 = \alpha_2 = 3$, $\beta_1 = \beta_2 = 2$, and $\delta(s) = s$. We note that, using this interpolation formula to specify the relative permeabilities, one can obtain the oil phase distribution in the invaded zone and match the calculation results for the hydrodynamic model with data of geophysical logging and laboratory tests of core samples.

During borehole drilling, the salt concentrations in the drilling mud and formation water are different; therefore, the filtrate invasion leads to a redistribution of the concentration and electrical resistance in the borehole zone [4]. To model the salt transport by the water phase, we use the transport equation for a conservative impurity [1]

$$\frac{\partial}{\partial t}(rmsc) = \frac{\partial}{\partial r}(-Fvc), \qquad c\Big|_{r=r_b} = c_{\rm w}, \qquad c\Big|_{t=0} = c_f, \tag{14}$$

where c is the relative concentration of the substance transported by the water phase and the functions $v(t)$ and $F(s)$ are defined in (9) and (10), respectively. The influence of hydrodynamic dispersion is considered insignificant. The solution of the hyperbolic equation (14) has the form of a stepped function: before the passage of the front, $c = c_{\rm w}$ for $r \leqslant l_c(t)$, and after the passage of the front, $c = c_f$ ($c_{\rm w}$ and c_f are the relative concentrations of the filtrate and formation water, respectively). Performing the normalization $\tilde{c} = (c - c_f)/(c_{\rm w} - c_f)$ (below, the tilde sign is omitted), we obtain $c = 1$ for $r < l_c(t)$ and $c = 0$ for $r > l_c(t)$.

Let us determine the equation of the front $r = l_c(t)$. Integrating Eq. (14) first over r from r_b to L and then over t from 0 to t taking into account the boundary and initial conditions and the form of the function c we obtain

$$\int_{r_b}^{l_c} rms\,dr = \int_0^t v(t)\,dt = V(t). \tag{15}$$

The value of $r = l_c$ is found from Eq. (15) using an algorithm similar to the algorithm used to find the dependence $l_s(t)$ from Eq. (13).

Estimation of the Parameters l_s and l_c. The ratio of the coordinates of the water-saturation and concentration fronts is one of the factors determining the structure of the invaded zone. This structure can easily be identified by electromagnetic sounding of oil-filled formations [6, 7]. We determine in what cases the condition $l_s(t) > l_c(t)$ is satisfied. As noted above, the positions of the fronts $l_s(t)$ and $l_c(t)$ is determined from relations (13) and (15), respectively. Relation (13) can be written as

$$\int_{r_b}^{l_s} rms\, dr = \int_{r_b}^{l_s} rms_f\, dr + (1 - F(s_f))V(t) = ms_f\,\frac{l_s^2 - r_b^2}{2} + (1 - F(s_f))V(t). \tag{16}$$

If $l_s(t) > l_c(t)$, then $\int_{r_b}^{l_s} rms\, dr > \int_{r_b}^{l_c} rms\, dr$. Then, relations (13) and (16) imply that

$$0 < -F(s_f)V(t) + ms_f(l_s^2 - r_b^2)/2. \tag{17}$$

Using the known relation at the displacement front $r = l_s$ [3], condition (17) can be written as

$$\frac{F(s_f)}{s_f} < \frac{F(s^-)}{s^-}, \qquad s^- = \lim_{r\to(l_s-0)} s.$$

If the water-saturation s has no discontinuity at the front, relation (17) leads to

$$\frac{dF}{ds}\Big|_{s(l_s)} > \frac{F(s_f)}{s_f}.$$

Calculations showed that, for real formations with a high oil saturation ($s_f < 0.5$), the salt concentration front lags behind the water-saturation front. Thus, in Fig. 1a, for curves 1, we have $l_s = 0.330$ and $l_c = 0.288$, for curves 2, $l_s = 0.351$ and $l_c = 0.291$, and for curves 3, $l_s = 0.329$ and $l_c = 0.286$. The same results were obtained by geophysical logging. It should be noted that capillary forces and hydrodynamic dispersion, as a rule, lead to smearing of oil-saturation and salt-concentration fronts. Nevertheless, this estimate provides a general understanding of the fluid distribution pattern in the invaded zone.

Accounting for Filter Cake Formation. Let us consider the boundary conditions (5) and (6) for the filter cake growth in greater detail. We note that, for $r = r_b$, relation (9) leads to $v(t) = -r_b k_{\mathrm{w}}(1)(\partial p/\partial r)\Big|_{r=r_b}$.

Denoting $Q(t) = \int_0^t q(t)\, dt$, we have

$$v(t) = r_b q(t), \qquad V(t) = r_b Q(t). \tag{18}$$

Equation (6) implies the relation $d' = \alpha(1 - d/d_{\max})^n Q'$, where the prime denotes the total time derivative. Integrating this equality and determining the integration constant from the initial conditions, we find the filter cake thickness d:

$$d = d_{\max} - d_{\max}\Big(\frac{\alpha(n-1)}{d_{\max}}\,Q + 1\Big)^{1/(1-n)}. \tag{19}$$

We note that, after d reaches the maximum value $d_{\max}$, the filter cake growth stops. Substituting (19) into (5), we obtain

$$\frac{dQ}{dt} = -\Big(\beta_0^{-1} + \frac{d_{\max}}{k_g^0}\Big(1 - \Big(\frac{\alpha(n-1)}{d_{\max}}\,Q + 1\Big)^{1/(1-n)}\Big)\Big)^{-1}\Big(p\Big|_{r=r_b} - p_b(t)\Big),$$

whence it follows that

$$\frac{d}{dt}\Big(Q\beta_0^{-1} + \frac{Qd_{\max}}{k_g^0} - \frac{d_{\max}^2}{k_g^0\alpha(n-1)}\,\frac{1-n}{2-n}\Big(\frac{\alpha(n-1)}{d_{\max}}\,Q + 1\Big)^{(2-n)/(1-n)}\Big) = -\Big(p\Big|_{r=r_b} - p_b(t)\Big). \tag{20}$$

Let us determine the pressure on the borehole boundary. Integrating (9) and determining the integration constant from condition (7), we obtain

$$p(t,r) = v(t) \int_r^D \frac{dr}{r(k_{\mathrm{w}}(s) + k_{\mathrm{oil}}(s))}. \tag{21}$$

Let us find an approximate solution of the problem considered. Usually, the depth of the invaded zone in which there was a change in the formation fluid distribution does not exceed 0.4–0.6 m, i.e., it is much smaller than the radius of influence L, which varies from a few tens to several hundreds of meters. As result, the formation permeability in the invaded zone has an insignificant effect on the pressure distribution. Hence, with high accuracy, for t $r = r_b$ the following relation holds:

$$p(t, r_b) = v(t) \int_{r_b}^L \frac{dr}{r(k_{\mathrm{w}}(s) + k_{\mathrm{oil}}(s))} \approx v(t) \int_{r_b}^L \frac{dr}{r(k_{\mathrm{w}}(s_f) + k_{\mathrm{oil}}(s_f))}.$$

Substituting the above relation into (20) and integrating over time from 0 to t [in view of (18)], we obtain the following equation for the approximate estimation of the filtrate volume $V(t)$:

$$\Big(\frac{\beta_0^{-1}}{r_b} + \frac{d_{\max}}{k_g^0 r_b} + \frac{\ln{(L/r_b)}}{k_{\mathrm{w}}(s_f) + k_{\mathrm{oil}}(s_f)}\Big)V + \frac{d_{\max}^2}{k_g^0 \alpha (2-n)} \Big(\frac{\alpha(n-1)}{d_{\max} r_b} V + 1\Big)^{(2-n)/(1-n)} = \int_0^t p_b(t)\, dt + \frac{d_{\max}^2}{k_g^0 \alpha (2-n)}.$$

Pressure Determination. In the numerical solution given above, the water-saturation distribution s was determined as a function of the radius r for a specified filtrate volume $V(t)$. According to (21), to determine the pressure accurately, it is necessary to express the total circulation rate $v(t)$ in terms of the filtrate volume $V(t)$. For this, we use the definition (9) for $r = r_b$ and boundary conditions (5)–(7) for $\beta_0^{-1} = 0$ and $n = 0$:

$$v(t) = -\frac{k_g^0 r_b^2}{\alpha V(t)} \Big(p\Big|_{r=r_b} - p_b(t)\Big).$$

Using expression (21) for the pressure p for $r = r_b$, we obtain

$$v(t) = \frac{k_g^0 r_b^2}{\alpha V(t)}\, p_b(t) \Big(1 + \frac{k_g^0 r_b^2}{\alpha V(t)} \int_{r_b}^L \frac{dr}{r(k_{\mathrm{w}}(s) + k_{\mathrm{oil}}(s))}\Big)^{-1}.$$

Then,

$$p(t,r) = \frac{k_g^0 r_b^2}{\alpha V(t)}\, p_b(t) \Big(1 + \frac{k_g^0 r_b^2}{\alpha V(t)} \int_{r_b}^L \frac{dr}{r(k_{\mathrm{w}}(s) + k_{\mathrm{oil}}(s))}\Big)^{-1} \int_r^L \frac{dr}{r(k_{\mathrm{w}}(s) + k_{\mathrm{oil}}(s))}.$$

Figure 2 shows curves of the pressure p versus radius r for $r_b = 0.1$ m, $\mu_0 = 0.4$, $m = 0.2$, $p_b = 400$ m, $L = 100$ m, $k_g = 0.001$ millidarcy, $\alpha = 0.08$, and $s_f = 0.2$. From Fig. 2 it follows that, for fixed formation permeability, the curves approach the abscissa as the filtrate volume increases. In formations with different permeabilities, the pressure distribution differs significantly: in well-permeable formations, the pressure near the borehole decreases rapidly, and in poorly permeable strata, its value remains very high for a fairly long time. This fact is important for the formation of vertical transport flows between neighboring strata with different permeabilities.

The algorithms given in this paper provide highly accurate solutions of the axisymmetric problem. Knowing only the filtrate volume $V(t)$, one can find all the basic characteristics of the hydrodynamic processes determining the formation of the invaded zone. This makes it possible to effectively solve the inverse problems of layer-by-layer hydrodynamic interpretation of oil reservoirs, in which an important required parameter is the filtrate volume [4, 6].

Time Determination. To find the relationship between borehole drilling conditions and the influence on the formation, it is necessary to know the time in which a definite filtrate volume enters the invaded zone. The dependence of the filtrate volume on time is found by solving the evolutionary difference problem. For the filtrate volume, the difference equation is written as

$$\frac{dV}{dt} \approx \frac{V_{j+1} - V_j}{\tau_j} = v_{j+1/2} = \frac{v_{j+1} + v_j}{2},$$

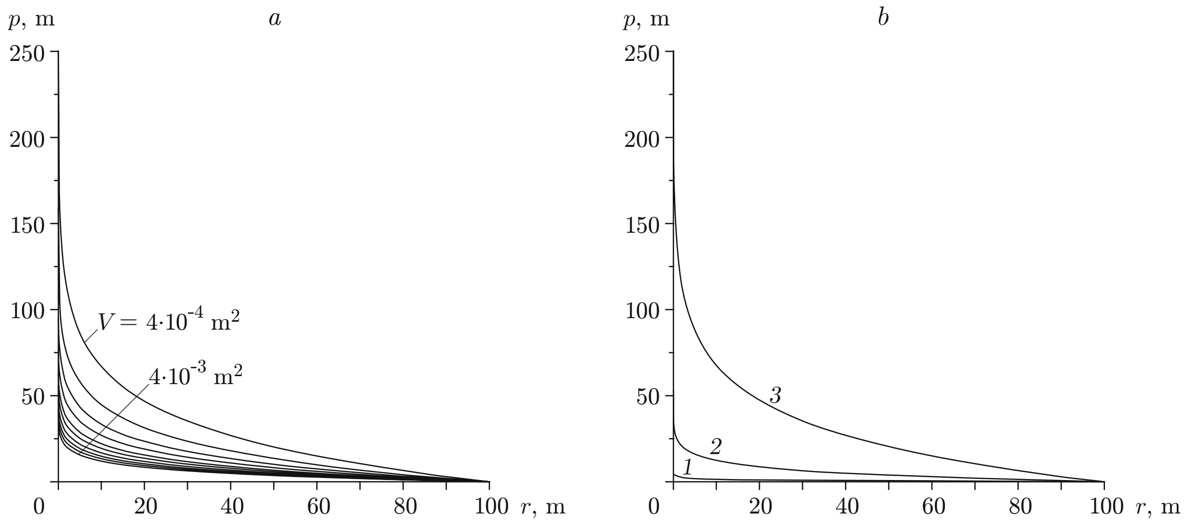

Fig. 2. Pressure versus radius: (a) $k = 1$ millidarcy and $\Delta V = 4 \cdot 10^{-4}$ m^2; (b) $V(t) = 4 \cdot 10^{-4}$ m^2 and $k = 100$ (1), 10 (2), and 1 millidarcy (3).

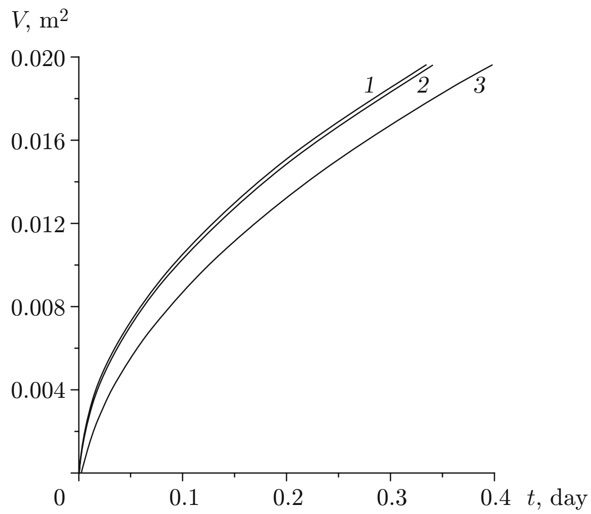

Fig. 3. Filtrate volume versus time for various formation permeabilities: $k = 100$ (1), 10 (2), and 1 millidarcy.

where j is the time step number and V_j and v_j are the values of the corresponding grid functions at the time $t_j = \sum\limits_{k=1}^{j} \tau_k$.

Figure 3 gives curves of filtrate volume versus time for various formation permeabilities ($r_b = 0.1$ m, $\mu_0 = 0.4$, $m = 0.2$, $p_b = 400$ m, $L = 100$ m, $s_f = 0.2$, $\alpha = 0.08$, $k_g = 0.005$ millidarsy). It is evident that the rate of increase in the filtrate volume, and, hence, the filter cake thickness has the maximum rate at the initial moment of formation drilling, after which this rate decreases. In addition, the difference in filtrate volume between well permeable and moderately permeable formations is insignificant; the filtrate volume decreases considerably only in poorly permeable formations.

Comparison with Experimental Data. Some characteristics of the model in question can be determined from geophysical logging measurements [4, 6]. The parameters of the model were specified using data on the geological structure of reservoirs and geophysical logging results for various boreholes. Caliper measurements yield the inner diameter of a borehole, from which the filter cake thickness d can be determined. In addition, one can use results electromagnetic sounding of boreholes, in which the electrical resistance in the near-borehole zone is measured on the basis of inversion of logs in the multipurpose system of high-frequency induction isoparametric

TABLE 1

Initial Parameters (Experiment)

Stratum number	z, m	$s_{\text{oil}}\|_{t=0}$	t, h	m	k_0, m/day
1	2519.4–2520.6	0.7045	39.63	0.1964	0.01989
2	2520.6–2521.4	0.7352	39.30	0.2055	0.04352
3	2521.4–2523.2	0.7748	38.87	0.2081	0.05497
4	2524.6–2526.0	0.6559	37.87	0.1915	0.01295
5	2526.0–2529.2	0.5988	37.10	0.1777	0.00422
6	2532.2–2533.0	0.6598	35.43	0.1964	0.01989
7	2535.2–2536.0	0.7606	34.43	0.2109	0.07048
8	2538.2–2539.6	0.5970	33.33	0.1955	0.01844
9	2539.6–2540.2	0.5236	33.00	0.1798	0.00468

TABLE 2

Experimental Data and Numerical Simulation Results

Stratum number	d, m		l_s, m (calculation)	r_2, m (experiment)	V, m^2 (calculation)
	Calculation	Experiment			
1	0.0087	0.0088	0.67	0.46	0.0160
2	0.0087	0.0088	0.65	0.73	0.0159
3	0.0086	0.0086	0.62	0.71	0.0158
4	0.0083	0.0084	0.70	0.56	0.0154
5	0.0080	0.0080	0.71	0.61	0.0151
6	0.0085	0.0090	0.67	0.65	0.0150
7	0.0080	0.0087	0.58	0.49	0.0149
8	0.0078	0.0078	0.67	0.49	0.0145
9	0.0075	0.0075	0.65	0.40	0.0141

logging [6]. The position of the boundary of the resistance variation region can be compared with the position of the oil displacement front in the hydrodynamic model, i.e., the calculated value of $r = l_s$ can be compared with the corresponding experimental value of r_2. Below, we give results of a comparison of numerical solutions and experimental data on these two parameters for a prospecting borehole in the West-Siberian oil deposit. The initial data and the results obtained are given in Tables 1 and 2. In addition, in the numerical calculations of the processes in this borehole, we used the following values of the problem parameters, which were not determined experimentally: $k_g = 0.003$ millidarcy, $\mu_0 = 0.4$, $n_{\text{w}} = n_{\text{oil}} = 3$, formation fluid density $\rho_f = 1.055 \cdot 10^{-3}$ kg/m^3, drilling mud density $\rho_s = 1.065 \cdot 10^{-3}$ kg/m^3, volumetric concentration of solid particles in the drilling mud $n_s = 0.034$ and filter cake porosity $m_g = 0.4$, $\alpha = n_s/[(1-n_s)(1-m_g)]$. The drilling rate is about 72 m/day, the borehole pressure in drilling is $p_b = \gamma z$ (z is the location of the stratum considered and $\gamma = (\rho_s - \rho_f)/\rho_{\text{w}} + \gamma_0$; $\gamma_0 = 0.14$; $\rho_{\text{w}} = 10^{-3}$ kg/m^3 is the water density). The parameter γ takes into account the excess of the borehole hydrostatic pressure over the formation hydrostatic pressure and the dynamic pressure losses due to drilling mud circulation.

From Tables 1 and 2, it follows that in the model proposed, the filter cake thickness is determined with high accuracy for all strata considered. In comparing the position of the oil displacement front in the hydrodynamic model and the position of the boundary of the resistance variation region, it is necessary to take into account factors that worsen the agreement between experimental and calculated values. First, in electromagnetic sounding, the positions of the strata adjoining clays in the upper part of the reservoir (stratum 1 in Tables 1 and 2) and strata with low oil saturations (strata 8 and 9 in Tables 1 and 2) inaccurately determine the depth of the invaded zone. Second, flows between layers with different permeabilities cause a redistribution of the filtrate and formation fluids in the stratum. Nevertheless, numerical and experimental data are generally in good agreement.

Conclusions. The one-dimensional axisymmetric problem of the displacement of formation fluids by a drilling mud filtrate in straight borehole drilling was considered in a general formulation taking into account the formation of a filter cake. The relations obtained can be used to solve inverse problems and perform studies based on spatial models.

The author thanks A. A. Kashevarov for help in the work.

Work was supported by an Integration project of the Siberian Division of the Russian Academy of Sciences (No. 75).

REFERENCES

1. *Development of Research into Filtration Theory in the USSR* [in Russian], Nauka, Moscow (1969).
2. R. E. Collins, *Flow of Fluids through Porous Materials*, Reinhold, New York (1961).
3. V. M. Entov and A. F. Zazovskii, *Hydrodynamics of Enhanced Oil Recovery Processes* [in Russian], Nedra, Moscow (1989).
4. A. A. Kashevarov, I. N. El'tsov, and M. I. Épov, "Hydrodynamic model for the evolution of an invaded zone in borehole drilling," *J. Appl. Mech. Tech. Phys.*, **44**, No. 6, 872–879 (2003).
5. B. T. Zhumagulov, N. V. Zubov, V. N. Monakhov, and Sh. S. Smagulov, *New Computer Technologies in Petroleum Production*, Gylym, Alma-Ata (1966).
6. I. N. El'tsov, A. A. Kashevarov, A. Yu. Sobolev, et al., "Evolution of an invaded zone from data of electromagnetic survey and hydrodynamic modeling," *Geologiya Geofizika*, **45**, No. 8, 97–108 (2004).
7. I. N. El'tsov, A. A. Kashevarov, A. Yu. Sobolev et al., "Generalization of Archie's formula and types of radial specific electric distribution in the near-borehole zone," *Geofiz. Vestn.*, No. 7, 9–14 (2004).